A new computer-based technique for automatic 3-D shape measurement is proposed and verified by experiments. In contrast to the moire contouring technique, a grating pattern projected onto the object surface is Fourier-transformed and processed in its spatial frequency domain as well as in its space-signal domain. This technique has a much higher sensitivity than the conventional moire technique and is capable of fully automatic distinction between a depression and an elevation on the object surface. There is no requirement for assigning fringe orders and interpolating data in the regions between contour fringes. The technique is free from errors caused by spurious moire fringes generated by the higher harmonic components of the grating pattern.

I. Introduction

The method of moire contouring is a well-known technique for 3-D shape measurement.\textsuperscript{1} Recent interest in the technique has been an automatic measurement based on data processing by computer.\textsuperscript{2-4} For this purpose it is essential to have means to (1) make automatic distinction between a depression and an elevation from a contour map of the object, (2) assign fringe orders automatically including those separated by discontinuities, (3) locate the center lines of broad fringes by correcting unwanted irradiance variations caused by nonuniform light reflection on the object surface, and (4) interpolate the regions lying between the contour lines. To satisfy these requirements, Ide-sawa et al.\textsuperscript{2} proposed the scanning moire method, and Moore and Truax\textsuperscript{4} proposed the phase-locked moire method. Further, we proposed another method called Fourier-transform profilometry (FTP) which is better for automatic measurement by computer processing.\textsuperscript{5} The idea of the FTP stemmed from the observation that all these cumbersome requirements mentioned above arise merely from attempting computer-based automatic measurement by means of a moire contouring technique that was originally developed for fringe analysis by human observation rather than by computer processing. Since FTP does not use moire fringes, it is free from all the difficulties associated with the moire contouring technique. Another great advantage of FTP is that it has a much higher sensitivity than the conventional moire technique. It can detect a shape variation much less than one contour fringe in moire topography. Our previous paper\textsuperscript{5} described a general principle applicable to both profilometry and interferometry, but experiments were given only for interferometry. The purpose of this paper is to give a more specific description of the principle of FTP and to present experimental results of 3-D shape measurement by FTP.

II. Optical Geometry

Optical geometry is similar to that of projection moire topography,\textsuperscript{6,7} but in FTP the grating image projected on an object surface is put directly into the computer and processed without using the second grating to generate moire fringes. Two different optical geometries have been proposed and used in moire topography; one has some merit over the other as well as some de-merit. In crossed-optical-axes geometry, the optical axes of a projector and a camera lie in the same plane and intersect a point near the center of the object. This geometry is easy to construct because both a grating and an image sensor can be placed on the optical axes of the projector and the camera, respectively, but it gives planar contours only when the optics are telecentric.\textsuperscript{8,9} In parallel-optical-axes geometry, the optical axes of a projector and a camera lie in the same plane and are parallel. This geometry gives planar contours but is somewhat awkward because the grating must be placed far off the optical axis of the projector to ensure that the grating image is formed within the field of view of the observation camera.\textsuperscript{2,6,7} These two options of optical geometry are also available in FTP, and in addition FTP...
can solve the problem of nonplanar contours in crossed-optical-axes geometry.

A. Crossed-Optical-Axes Geometry

Figure 1 shows a geometry in which the optical axes $E_p E_p$ of a projector lens crosses the other optical axis $E_c E_c$ of a camera lens at point $O$ on a reference plane $R$, which is a fictitious plane normal to $E_c E_c$ and serves as a reference from which object height $h(x,y)$ is measured. Grating $G$ has its lines normal to the plane of the figure, and its conjugate image (with period $p$) is formed by the projector lens on plane $I$ through point $O$; $E_p$ and $E_c$ denote, respectively, the centers of the entrance and the exit pupils of the projector lens. The camera lens, with the centers of the entrance and the exit pupils at $E_p$ and $E_c$, images reference plane $R$ onto the image sensor plane $S$. $E_p$ and $E_c$ are located at the same distance $l_0$ from plane $R$. It should be noted that $E_p$ and $E_c$ are the centers of the pupils, not the nodal points of the lenses as is so often confused in the literature. When the object is a flat and uniform plane on $R$, i.e., $h(x,y) = 0$, and if $E_p$ is at infinity (as denoted by $E$ for a telecentric projector), the grating image projected on the object surface and observed through $E_c$ is a regular grating pattern which can be expressed by a Fourier series expansion:

$$g_0(x,y) = \sum_{n=-\infty}^{\infty} A_n \exp\{2\pi i n/p\}, \quad (1)$$

where $s_0 = BC$ is a function of $x$ and has a positive sign when $C$ is to the right of $B$ as in the figure. For the convenience of later discussion, we express Eq. (3) as a spatially phase-modulated signal

$$g_0(x,y) = \sum_{n=-\infty}^{\infty} A_n \exp\{2\pi i n/\phi_0(x) + n\phi_0(x)\}, \quad (4)$$

where

$$\phi_0(x) = 2\pi f_0 s_0(x) = 2\pi s_0(BC). \quad (5)$$

Since the grating image is deformed and phase-modulated even for $h(x,y) = 0$, the crossed-optical-axes geometry, when used in moire topography, gives nonplanar contours unless the pupils are at infinity, i.e., the case of telecentric optics. This has imposed a great restriction on the application of the nontelecentric crossed-optical-axes geometry to moire topography, in spite of its easy-to-construct merit. In FTP, this initial phase modulation is automatically corrected as will be shown in the next section.

For a general object with varying $h(x,y)$, the principal ray $E_p A$ strikes the object surface at point $H$, and point $H$ will be seen to be a point $D$ on plane $R$ when observed through $E_c$. Hence, the deformed grating image for a general object is given by

$$g(x,y) = r(x,y) \cdot \sum_{n=-\infty}^{\infty} A_n \exp\{2\pi i n/\phi(x) + n\phi(x)\}, \quad (6)$$

or

$$g(x,y) = r(x,y) \cdot \sum_{n=-\infty}^{\infty} A_n \exp\{2\pi i n/\phi(x) + n\phi(x)\}, \quad (7)$$

where

$$\phi(x,y) = 2\pi f_0 s(x,y) = 2\pi s_0 BD, \quad (8)$$

and $r(x,y)$ is a nonuniform distribution of reflectivity on the object surface.

B. Parallel-Optical-Axes Geometry

Figure 2 shows a geometry in which the optical axis $E_p E_p$ of a projector lens and that of a camera lens $E_c E_c$ are parallel and are normal to reference plane $R$. The conjugate image of grating $G$ is formed on plane $R$, and the three points $A$, $B$, and $C$ in Fig. 1 degenerate into point $C$ in Fig. 2, so that Eqs. (5) and (8) become...
0 is now subtracted, $A\phi(x, y)$ in Eqs. (16) and (17) gives

$$\phi_0(x) = 2\pi f_0\phi_0(x) = 2\pi f_0\vec{B}\vec{C} = 0, \quad (9)$$
$$\phi(x, y) = 2\pi f_0\phi(x, y) = 2\pi f_0\vec{C}\vec{D}. \quad (10)$$

Hence, the grating image projected on the plane $h(x, y)$ = 0 remains as a regular grating pattern regardless of the position of the pupil of the projector. This brings a merit of generating planar contours in moire topography, but the optical setup becomes more or less awkward because the grating must be translated far off the optical axis of the projector lens in order to form its image within the field of view of the camera lens.

### III. Fourier Transform Method

The deformed grating image given by Eq. (7) can be interpreted as multiple signals with spatial carrier frequencies $nf_0$ modulated both in phase $\phi(x, y)$ and amplitude $r(x, y)$. Since the phase carries information about the 3-D shape to be measured, the problem is how to obtain $\phi(x, y)$ separately from the unwanted amplitude variation $r(x, y)$ caused by nonuniform reflectivity on the object surface. We rewrite Eq. (7) as

$$g(x, y) = \sum_{n=-\infty}^{\infty} q_n(x, y) \cdot \exp(2\pi inf_0x), \quad (11)$$

where

$$q_n(x, y) = A_n r(x, y) \cdot \exp(in\phi(x, y)). \quad (12)$$

By using a FFT algorithm, we compute the 1-D Fourier transform of Eq. (12) for the variable $x$ only, with $y$ being fixed:

$$G(f, y) = \sum_{x=-\infty}^{\infty} g(x, y) \exp(-2\pi i fx) dx \approx \sum_{x=-\infty}^{\infty} q_n(f - n/fo) \cdot \exp(-2\pi i fx) dx, \quad (13)$$

where $G(f, y)$ and $Q_n(f, y)$ are the 1-D Fourier spectra of $g(x, y)$ and $q_n(x, y)$, respectively, computed with respect only to the variable $x$, and the other variable $y$ being treated as a fixed parameter. Since in most cases $r(x, y)$ and $\phi(x, y)$ vary very slowly compared to the frequency $fo$ of the grating pattern, all the spectra $Q_n(f - n/fo)$ are separated from each other by the carrier frequency $fo$, as shown in Fig. 3. We select only one spectrum $Q_1(f - fo, y)$ dotted in the figure and compute its inverse Fourier transform to obtain a complex signal

$$\hat{g}(x, y) = q_1(x, y) \cdot \exp(2\pi if_0x) = A_1 r(x, y) \cdot \exp(2\pi if_0x + \phi(x, y)). \quad (14)$$

In the crossed-optical-axes case, we do the same filtering operation for Eq. (4) to obtain

$$\hat{g}_o(x, y) = A_1 \exp[2\pi if_0x + \phi(x, y)], \quad (15)$$

and generate from Eqs. (14) and (15) a new signal

$$\hat{g}(x, y) \cdot \hat{g}_o(x, y) = |A_1|^2 r(x, y) \cdot \exp[\Delta\phi(x, y)], \quad (16)$$

where

$$\Delta\phi(x, y) = \phi(x, y) - \phi_0(x) = 2\pi f_0(BD - BC) = 2\pi f_0\vec{C}\vec{D}. \quad (17)$$

Since the initial phase modulation $\phi_0(x)$ for $h(x, y)$ = 0 is now subtracted, $\Delta\phi(x, y)$ in Eqs. (16) and (17) gives

$$\phi_0 = 2\pi f_0\phi_0 \tan(\delta\alpha), \quad (18)$$

which gives rise to a mismatching error in moire topography. We therefore use the formulas of Eqs. (16) and (17) in both cases.

Now our task is to obtain the phase distribution $\Delta\phi(x, y)$ in Eq. (16), separating it from the unwanted amplitude variation $r(x, y)$. Noting that both $|A_1|^2 \cdot r(x, y)$ and $\Delta\phi(x, y)$ in Eq. (16) are real functions, we compute a complex logarithm of Eq. (16):

$$\log[|g(x, y)| \cdot e^{i\phi(x, y)}] = \log[|A_1|^2 r(x, y)] + i\Delta\phi(x, y). \quad (19)$$

We obtain the phase distribution $\Delta\phi(x, y)$ in the imaginary part, completely separated from the unwanted variation of reflectivity $r(x, y)$ in the real part. Since the phase calculation by computer gives principal values ranging from $-\pi$ to $\pi$, the phase distribution is wrapped into this range and consequently has discontinuities with $2\pi$-phase jumps for variations more than $2\pi$. These discontinuities can be corrected easily by adding or subtracting $2\pi$ according to the phase jump ranging from $\pi$ to $-\pi$ or vice versa. A 2-D phase distribution can be obtained simply by repeating the same procedure for the $y$ sections. Details of the phase-unwrapping algorithm are described in Ref. 5. Since the loci of the discontinuities with $2\pi$-phase jumps correspond to contour fringes in moire topography, the phase-unwrapping process plays the role of the fringe order assignment algorithm of conventional moire topography.

### IV. Phase-to-Height Conversion

In this section we derive a formula for converting the measured phase distribution into the physical height distribution. Noting that $\Delta\phi E_p HE_c \wedge \Delta\phi CDH$ in both Figs. 1 and 2, we can write

$$\vec{CD} = -dh(x, y)/[l_0 - h(x, y)], \quad (20)$$

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where the object height \( h(x,y) \) is defined positive when measured upward from reference plane \( R \). Substituting Eq. (20) into Eq. (17) and solving it for \( h(x,y) \), we obtain the conversion formula

\[
 h(x,y) = l_0 \Delta \phi(x,y)/[\Delta \phi(x,y) - 2\pi/\gamma d].
\]  

(21)

We can express this formula in another form which can be directly compared to the well-known formula of moire topography. Substituting Eq. (2) into Eq. (21), we have

\[
 h(x,y) = l_0 \Delta \phi(x,y)/[\Delta \phi(x,y) - 2\pi/\gamma d].
\]  

(22)

When we note that \( \Delta \phi(x,y)/2\pi \) gives the number \( n \) of the fringe order in moire topography, we see that Eq. (22) is exactly the same as the formula of moire topography. However, the difference should be noted that, whereas in the moire technique the height distribution information is given only along a discrete set of contour lines, our technique, FTP, gives the height information at all picture elements regardless of whether \( A_0(x,y)/2\pi \) is an integer or not. This is the reason that FTP does not need fringe interpolation as is necessary in moire topography.

V. Maximum Range of Measurement

Since FTP is based on filtering for selecting only a single spectrum of the fundamental frequency component, the carrier frequency \( f_0 \) must separate this spectrum from all other spectra. This condition limits the maximum range measurable by FTP. Noting that \( r(x,y) \) varies much slower than \( f_0 \) in Eq. (7), we define for the \( n \)th spectrum component a local spatial frequency \( f_n \), analogous to an instantaneous frequency of FM signal: 

\[
 f_n = \frac{1}{2\pi} \frac{\partial}{\partial x} \left[ 2\pi n f_0 + n \phi(x,y) \right] = n f_0 + \frac{n}{2\pi} \frac{\partial \phi(x,y)}{\partial x}.
\]  

(23)

For the fundamental spectrum to be separated from all other spectra, it is necessary that

\[
 (f_1)_{\text{max}} < (f_1)_{\text{min}}, \quad (n = 2,3,\ldots)
\]  

(24)

and that

\[
 f_b < (f_1)_{\text{min}},
\]  

(25)

where \( f_b \), \((f_n)_{\text{max}}\) and \((f_n)_{\text{min}}\) are shown in Fig. 4 for \( n = 1,2,3 \). Substituting Eq. (23) into Eqs. (24) and (25), we have

\[
 f_0 + \frac{1}{2\pi} \frac{\partial \phi}{\partial \gamma} < n f_0 + \frac{n}{2\pi} \frac{\partial \phi}{\partial \gamma},
\]  

\[
 (n = 2,3,\ldots),
\]  

(26)

\[
 f_b < f_0 + \frac{1}{2\pi} \frac{\partial \phi}{\partial \gamma},
\]  

(27)

A safer and more practical condition can be set by

\[
 f_0 + \frac{1}{2\pi} \frac{\partial \phi}{\partial \gamma} < n f_0 - \frac{n}{2\pi} \frac{\partial \phi}{\partial \gamma},
\]  

\[
 (n = 2,3,\ldots),
\]  

(28)

where \( \left| \frac{\partial \phi}{\partial \gamma} \right| \) denotes the maximum absolute value which is a larger value of \( \left| \frac{\partial \phi}{\partial \gamma} \right|_{\text{max}} \) and \( \left| \frac{\partial \phi}{\partial \gamma} \right|_{\text{min}} \). From Eqs. (28) and (29) we have

\[
 \frac{\partial \phi}{\partial \gamma} < \frac{n - 1}{n + 1} 2\pi f_0, \quad (n = 2,3,\ldots),
\]  

(30)

\[
 \frac{\partial \phi}{\partial \gamma} < 2\pi (f_0 - f_b).
\]  

(31)

Since in most cases \( f_b \) is much smaller than \( f_0/2 \) and \((n - 1)/(n + 1)\) increases monotonically with \( n \), the limit is set by Eq. (30) for \( n = 2 \):

\[
 \frac{\partial \phi}{\partial \gamma} < \frac{2\pi f_0}{3}.
\]  

(32)

When discussing the maximum range of measurement, we can assume that \( \phi(x,y) \) is much larger than \( \phi_0(x) \), and from Eq. (21) we can write

\[
 \phi(x,y) = \Delta \phi(x,y) = \left( 2\pi f_0 d/\gamma_0 \right) h(x,y),
\]  

(33)

where we have also assumed that \( l_0 > h(x,y) \). Substituting Eq. (33) into Eq. (32), we finally obtain

\[
 \frac{\partial h(x,y)}{\partial x} \left|_{\text{max}} \right. < \frac{1}{3} \frac{l_0}{d},
\]  

(34)

This condition states that the maximum range of measurement is not limited by the height distribution \( h(x,y) \) itself but by its derivative in the direction normal to the line of the grating. The maximum range of measurement can be extended by employing a geometry in which \( l_0/d \) is large to prevent the phase from being overmodulated. This corresponds to reducing the fringe sensitivity in the case of moire topography, but FTP retains a sensitivity sufficient for most applications since it can detect a phase distribution much less than \( 2\pi \). This will be demonstrated by experiments in Sec. VI.

VI. Experiments

Figure 5 shows a schematic diagram of the experimental setup. A crossed-optical-axes geometry was employed because of its easy-to-construct merit. A 300-W slide projector with an 85-mm focal length projecting lens was used to project a Ronchi grating of 150 lines/in. onto an object surface. The object is a whiskey bottle embedded in a uniform plane plate which serves
as a reference plane in the background. The deformed grating pattern was observed by a low distortion TV camera (Hamamatsu C-1000) with a 55-mm focal length Micro-Nikkor lens. An analog video output signal is converted into an 8-bit digital signal and stored in a frame memory in the form of a picture with 512 x 512 pixels which can be monitored through a TV monitor. The picture in the frame memory is DMA transferred to the memory of a Digital LSI-11/23 microcomputer to make a temporal file on a disk which is then transferred through a communication line to a faster Digital PDP-11/44 minicomputer and processed. The final result is sent back and displayed on an X-Y plotter. Figure 6 shows a picture of a deformed or phase-modulated grating pattern, where the straight grating lines in the background serve as reference signals for determining the absolute phase values to be converted into a height distribution. Figure 7 shows an example of the irradiance profile along a horizontal line in the direction of the x axis. Note that the reflectivity \( r(x,y) \) is strongly nonuniform over the object surface. Figure 8 shows Fourier spectra of Eq. (13) computed by using a FFT algorithm. In the figure, the large spectrum \((n = 0)\) is clipped to get an enhanced view of other spectra. Note that the spectrum \((n = 1)\) is completely separated from other spectra satisfying the condition of Eq. (34). Figure 9 shows a wrapped phase distribution \( \Delta \phi(x,y) \) computed from the imaginary part of Eq. (19). Noting
that from Eq. (22) the line along the discontinuities with $2\pi$-phase jumps corresponds to one contour fringe in moire topography, we can see that the height variation less than the amount of one fringe is clearly detected in the figure. Figure 10 shows the unwrapped phase distribution which has the form of a whiskey bottle. This phase distribution is converted into the height distribution using the formula of Eq. (21) and compared with the result of direct measurement by the contact method. Figure 11 shows three examples of the object profiles, where the lines and circles represent the results obtained by FTP and the contact method, respectively.

**VII. Conclusion**

We have proposed a new technique, Fourier transform profilometry, which is suitable for automatic measurement of a 3-D object shape. Since FTP does not use the moire contouring technique, it is completely free from various cumbersome problems associated with moire topography. For example, FTP can accomplish fully automatic distinction between a depression and an elevation of the object shape, it requires no fringe-order assignments or fringe-center determination, and it needs no interpolation between fringes as it gives height distribution at all the picture elements over the object image. Furthermore, FTP has the advantage that it can detect height variations less than the amount of one fringe in the conventional moire contouring technique. Another merit of FTP is that it is perfectly free from the effect of unwanted spurious moire fringes generated by the higher harmonic components of the grating pattern, since these components are filtered out in the spectrum domain. Finally, we discussed the applicability of FTP and proposed a practical criterion that the maximum slope of the object be less than $l_0/3d$, where $l_0$ and $d$ are the distances between the camera and the object and the camera and the projector, respectively.

**References**


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